3 Other Considerations

3.1 Categorical Predictors

What to do when X_i are categorical?

So far we have assumed all variables in our linear model are quantitiative.

For example, consider building a model to predict highway gas mileage from the `mpg` data set.

```
head(mpg)
```

```
## # A tibble: 6 x 11
##   manufacturer model displ  year   cyl trans      drv     cty   hwy fl    class
##   <chr>        <chr> <dbl> <int> <int> <chr>      <chr> <int> <int> <chr> <chr>
## 1 audi         a4      1.8  1999     4 auto(l5)   f        18    29 p     compa
## 2 audi         a4      1.8  1999     4 manual(m5) f        21    29 p     compa
## 3 audi         a4      2     2008     4 manual(m6) f        20    31 p     compa
## 4 audi         a4      2     2008     4 auto(av)   f        21    30 p     compa
## 5 audi         a4      2.8  1999     6 auto(l5)   f        16    26 p     compa
## 6 audi         a4      2.8  1999     6 manual(m5) f        18    26 p     compa
```

```
library(GGally)

mpg %>%
  select(-model) %>% # too many models
  ggpairs() # plot matrix
```

makes $\frac{p(p-1)}{2}$ plots to look at each pair of variables

looks at type of variables and chooses appropriate plot type.

To incorporate these categorical variables into the model, we will need to introduce $k-1$ dummy variables, where $k =$ the number of levels in the variable, for each qualitative variable.

For example, for `drv`, we have 3 levels: `4`, `f`, and `r`. $k=3$

$$x_{i1} = \begin{cases} 1 & \text{if } i^{th} \text{ car is front wd} \\ 0 & \text{if } i^{th} \text{ car is NOT front WD.} \end{cases}$$

$$x_{i2} = \begin{cases} 1 & \text{if } i^{th} \text{ car is RWD} \\ 0 & \text{if } i^{th} \text{ car is NOT RWD} \end{cases}$$

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \varepsilon_i = \begin{cases} \beta_0 + \beta_1 + \varepsilon_i & \text{if } i^{th} \text{ car is FWD} \\ \beta_0 + \beta_2 + \varepsilon_i & \text{if } i^{th} \text{ car is RWD} \\ \beta_0 + \varepsilon_i & \text{if } i^{th} \text{ car is 4WD} \end{cases} \Rightarrow$$

$\beta_0 =$ avg hwy mpg for 4WD cars

$\beta_1 =$ difference in average hwy mpg btw/ FWD & 4WD

$\beta_2 =$ difference in average hwy mpg btw/ RWD & 4WD.

```r
lm(hwy ~ displ + cty + drv, data = mpg) %>%
  summary()
```

R creates dummy variables for us.

```
##
## Call:
## lm(formula = hwy ~ displ + cty + drv, data = mpg)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.6499 -0.8764 -0.3001  0.9288  4.8632
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.42413    1.09313   3.132  0.00196 **
## displ       -0.20803    0.14439  -1.441  0.15100
## cty          1.15717    0.04213  27.466  < 2e-16 ***
## drvf         2.15785    0.27348   7.890 1.23e-13 ***
## drvr         2.35970    0.37013   6.375 9.95e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.49 on 229 degrees of freedom
## Multiple R-squared:  0.9384, Adjusted R-squared:  0.9374
## F-statistic: 872.7 on 4 and 229 DF,  p-value: < 2.2e-16
```

3.2 Extensions of the Model

The standard regression model provides interpretable results and works well in many problems. However it makes some very strong assumptions that may not always be reasonable.

* linear model
constant error variance
uncorrelated errors (w/ X).

Additive Assumption

The additive assumption assumes that the effect of each predictor on the response is not affected by the value of the other predictors. What if we think the effect should depend on the value of another predictor?

```
lm(sales ~ TV + radio + TV*radio, data = ads) %>%
  summary()
```

interaction term

$$Y = \beta_0 + \beta_1 X + \beta_2 X_2 + \beta_3 X_1 X_2 + \varepsilon$$

$$= \beta_0 + \underbrace{(\beta_1 + \beta_3 X_2)}_{\text{changes w/ } X_2 \text{ value}} X_1 + \beta_2 X_2 + \varepsilon$$

```
##
## Call:
## lm(formula = sales ~ TV + radio + TV * radio, data = ads)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.3366 -0.4028  0.1831  0.5948  1.5246
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 6.750e+00  2.479e-01  27.233   <2e-16 ***
## TV          1.910e-02  1.504e-03  12.699   <2e-16 ***
## radio       2.886e-02  8.905e-03   3.241   0.0014 **
## TV:radio    1.086e-03  5.242e-05  20.727   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9435 on 196 degrees of freedom
## Multiple R-squared:  0.9678, Adjusted R-squared:  0.9673
## F-statistic:  1963 on 3 and 196 DF,  p-value: < 2.2e-16
```

β_3 is significantly different from zero

significant relationship.

$R^2 = 0.89$ without the interaction
big increase ⇒ better fitting model

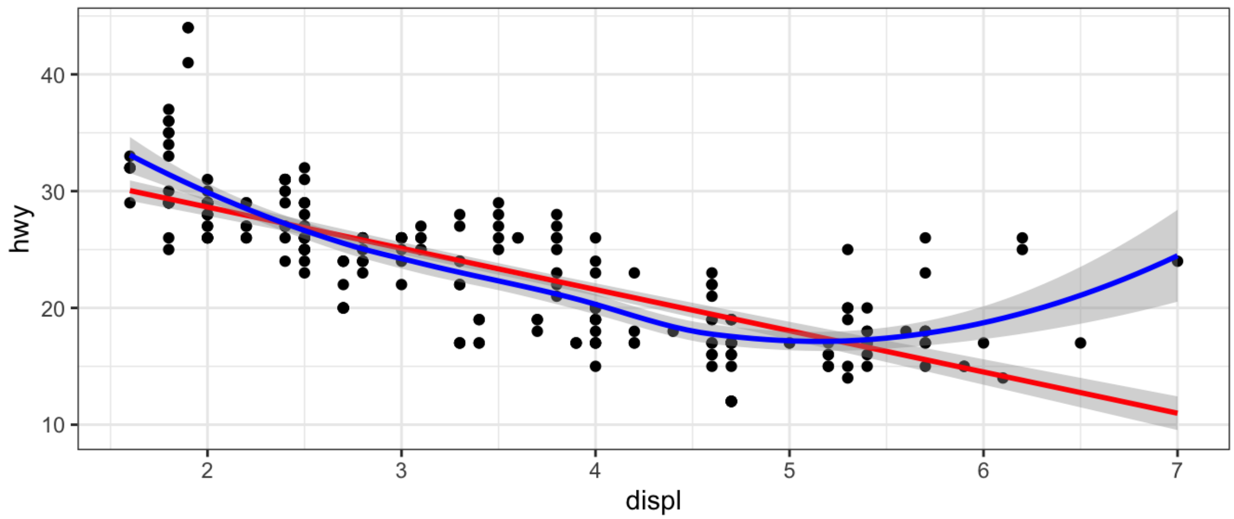
→ "an increase of \$1000 in radio advertising will be associated w/ an increase in sales of $(\hat{\beta}_2 + \hat{\beta}_3 \times TV) \times 1000$
29 + 1.1 × TV units.

WARNING: if we add interaction term be sure to keep original variables in the model, otherwise very confusing to interpret results.

Linearity Assumption

The linear regression model assumes a linear relationship between response and predictors. In some cases, the true relationship may be non-linear.

```
ggplot(data = mpg, aes(displ, hwy)) +
  geom_point() +
  geom_smooth(method = "lm", colour = "red") +
  geom_smooth(method = "loess", colour = "blue")
```



← may be non-linear

How to include nonlinear terms in model?

$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \varepsilon$

Y X X^2

```
lm(hwy ~ displ + I(displ^2), data = mpg) %>%
  summary()
```

identity function

```
##
## Call:
## lm(formula = hwy ~ displ + I(displ^2), data = mpg)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.6258 -2.1700 -0.7099  2.1768 13.1449
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  49.2450     1.8576  26.510  < 2e-16 ***
## displ       -11.7602     1.0729 -10.961  < 2e-16 ***
## I(displ^2)    1.0954     0.1409   7.773 2.51e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.423 on 231 degrees of freedom
## Multiple R-squared:  0.6725, Adjusted R-squared:  0.6696
## F-statistic: 237.1 on 2 and 231 DF,  p-value: < 2.2e-16
```

significant

WARNING: be careful throwing in higher level polynomial powers → will lead to overfitting & very bad prediction on the edges of the space.

3.3 Potential Problems

1. Non-linearity of response-predictor relationships

   diagnosis: plot residuals vs. ① fitted values → or vs. ② each predictor. see pattern.

   solution: - add polynomial term OR - transform predictor; - add interaction

2. Correlation of error terms

   diagnosis: understanding how data is collected. time series? spatial data?

   solution: use models formulated for those correlated errors (not this class).

3. Non-constant variance of error terms

   errors correlated w/ X

   diagnosis: plot residuals vs. fitted values. see funnel pattern.

   solution: transform Y – try log Y or $\sqrt{Y}$

4. Outliers

   diagnosis: plot data

   solution: is your data wrong? i.e. error in collection? fix it. otherwise – maybe you are missing a predictor? (robust statistics/regression)

4 K-Nearest Neighbors

In Ch. 2 we discuss the differences between *parametric* and *nonparametric* methods. Linear regression is a parametric method because it assumes a linear functional form for $f(X)$

- easy to fit
  easy to interpret
  can do hypothesis test

make strong assumptions, what if they are wrong?
→ parametric methods perform poorly.

A simple and well-known non-parametric method for regression is called K-nearest neighbors regression (KNN regression).

\# of neighbors

Given a value for K and a prediction point x_0, KNN regression first identifies the K training observations that are closest to x_0 ($\mathcal{N}_0$). It then estimates $f(x_0)$ using the average of all the training responses in $\mathcal{N}_0$,

training data

$$\hat{f}(x_0) = \frac{1}{K} \sum_{x_i \in \mathcal{N}_0} y_i$$

value to be predicted

```
library(caret) # package for knn
set.seed(445) #reproducibility
```

fake data

```
x <- rnorm(100, 4, 1) # pick some x values
y <- 0.5 + x + 2*x^2 + rnorm(100, 0, 2) # true relationship
df <- data.frame(x = x, y = y) # data frame of training data
```

k = 2, 4, 6, 8, 10

```
for (k in seq(2, 10, by = 2)) {
  knn_model <- knnreg(y ~ x, data = df, k = k) # fit knn model
```

formula just like lm — specify k

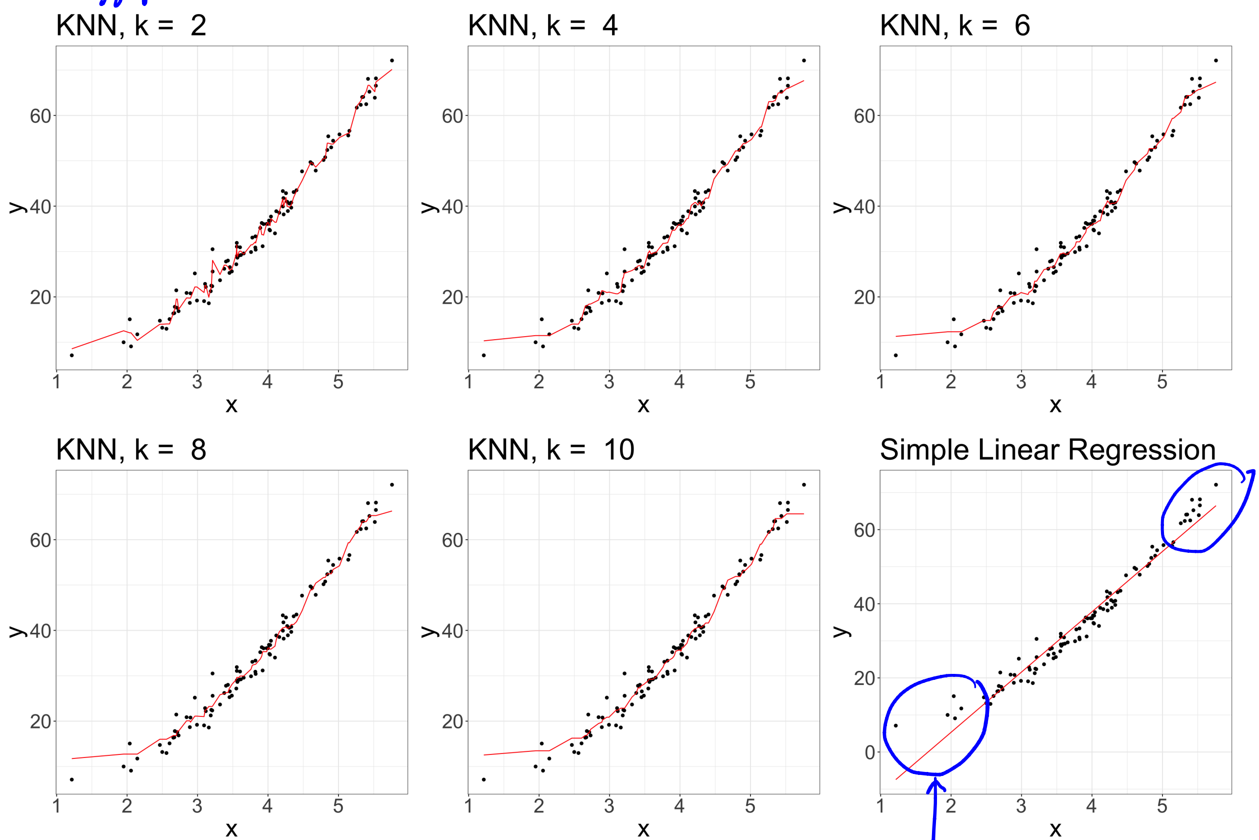
```
  ggplot(df) +
    geom_point(aes(x, y)) +
    geom_line(aes(x, predict(knn_model, df)), colour = "red") +
    ggtitle(paste("KNN, k = ", k)) +
    theme(text = element_text(size = 30)) -> p

  print(p) # knn plots
}
```

compare to linear model

```
ggplot(df) +
    geom_point(aes(x, y)) +
    geom_line(aes(x, lm(y ~ x, df)$fitted.values), colour = "red") +
    ggtitle("Simple Linear Regression") +
    theme(text = element_text(size = 30)) # slr plot
```

wiggly



smooth

missing non-linear nature of relationship

$k \uparrow \implies$ smoother fit.