3 LDA

Logistic regression involves direction modeling $P(Y = k|X = x)$ using the logistic function for the case of two response classes. We now consider a less direct approach.

Idea:

Why do we need another method when we have logistic regression?

1.

2.

3.

3.1 Bayes' Theorem for Classification

Suppose we wish to classify an observation into one of K classes, where $K \geq 2$.

π_k

$f_k(x)$

$P(Y = k|X = x)$

In general, estimating π_k is easy if we have a random sample of Y's from the population.

Estimating $f_k(x)$ is more difficult unless we assume some particular forms.

3.2 p = 1

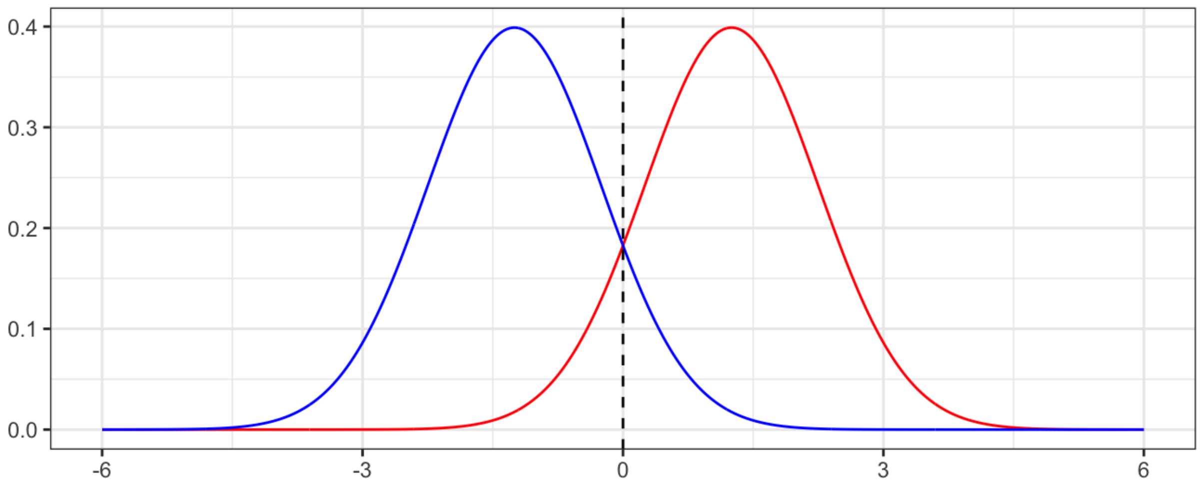
Let's (for now) assume we only have 1 predictor. We would like to obtain an estimate for $f_k(x)$ that we can plug into our formula to estimate $p_k(x)$. We will then classify an observation to the class for which $\hat{p}_k(x)$ is greatest.

Suppose we assume that $f_k(x)$ is normal. In the one-dimensional setting, the normal density takes the form

Plugging this into our formula to estimate $p_k(x)$,

We then assign an observation $X = x$ to the class which makes $p_k(x)$ tthe largest. This is equivalent to

Example 3.1 Let $K = 2$ and $\pi_1 = \pi_2$. When does the Bayes classifier assign an observation to class 1?

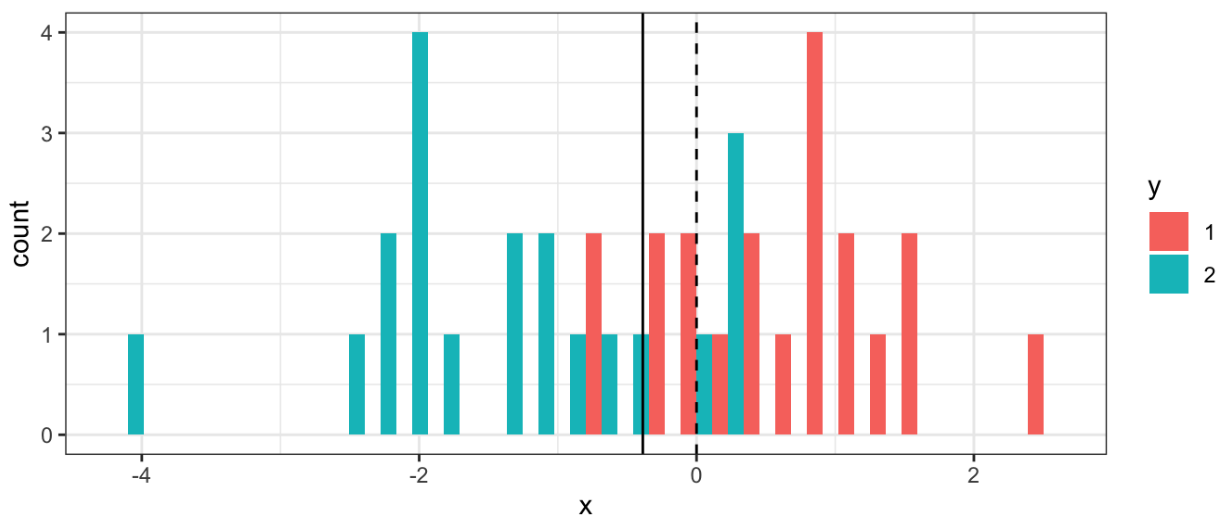


In practice, even if we are certain of our assumption that X is drawn from a Gaussian distribution within each class, we still have to estimate the parameters $\mu_1, \dots, \mu_K, \pi_1, \dots, \pi_K, \sigma^2$.

The *linear discriminant analysis* (LDA) method approximated the Bayes classifier by plugging estimates in for π_k, μ_k, σ^2.

Sometimes we have knowledge of class membership probabilities $\pi_1, \dots, \pi_K$ that can be used directly. If we do not, LDA estimates π_k using the proportion of training observations that belong to the kth class.

The LDA classifier assignes an observation $X = x$ to the class with the highest value of




```
##    pred
## y       1     2
##   1 18966  1034
##   2  3855 16145
```

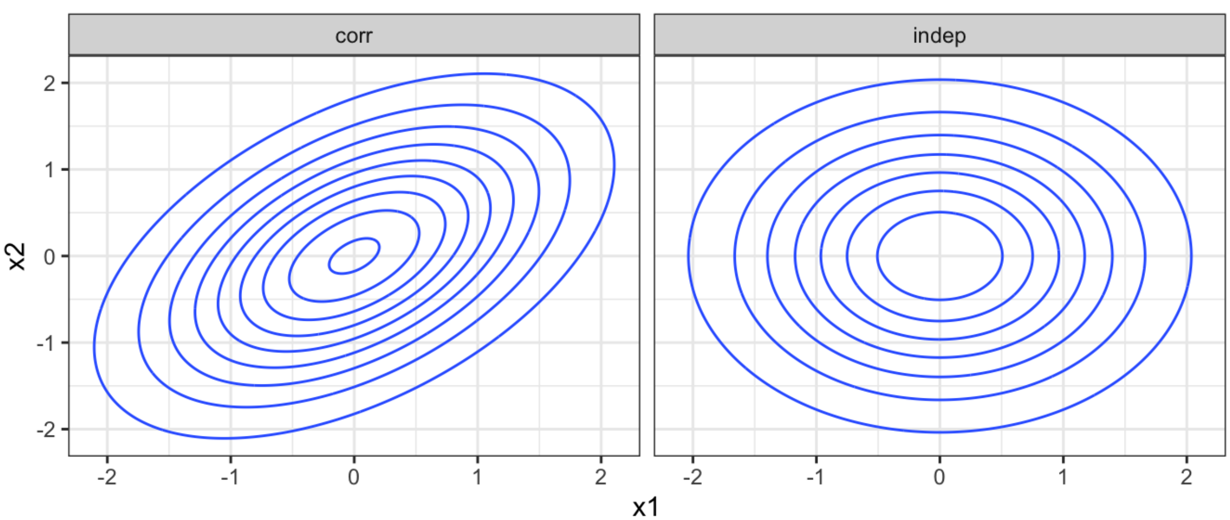
The LDA test error rate is approximately 12.22% while the Bayes classifier error rate is approximately 10.52%.

The LDA classifier results from assuming that the observations within each class come from a normal distribution with a class-specific mean vector and a common variance σ^2 and plugging estimates for these parameters into the Bayes classifier.

3.3 $p > 1$

We now extend the LDA classifier to the case of multiple predictors. We will assume

Formally the multivariate Gaussian density is defined as



In the case of $p > 1$ predictors, the LDA classifier assumes the observations in the kth class are drawn from a multivariate Gaussian distribution $N(\boldsymbol{\mu}_k, \boldsymbol{\Sigma})$.

Plugging in the density function for the kth class, results in a Bayes classifier

Once again, we need to estimate the unknown parameters $\boldsymbol{\mu}_1, \dots, \boldsymbol{\mu}_K, \pi_1, \dots, \pi_K, \boldsymbol{\Sigma}$.

To classify a new value $X = x$, LDA plugs in estimates into $\delta_k(x)$ and chooses the class which maximized this value.

Let's perform LDA on the `Default` data set to predict if an individual will default on their CC payment based on balance and student status.

```r
library(MASS) # package containing lda function
lda_fit <- lda(default ~ student + balance, data = Default)
lda_fit
```

```
## Call:
## lda(default ~ student + balance, data = Default)
##
## Prior probabilities of groups:
##     No    Yes
## 0.9667 0.0333
##
## Group means:
##     studentYes   balance
## No   0.2914037  803.9438
## Yes  0.3813814 1747.8217
##
## Coefficients of linear discriminants:
##                     LD1
## studentYes -0.249059498
## balance     0.002244397
```

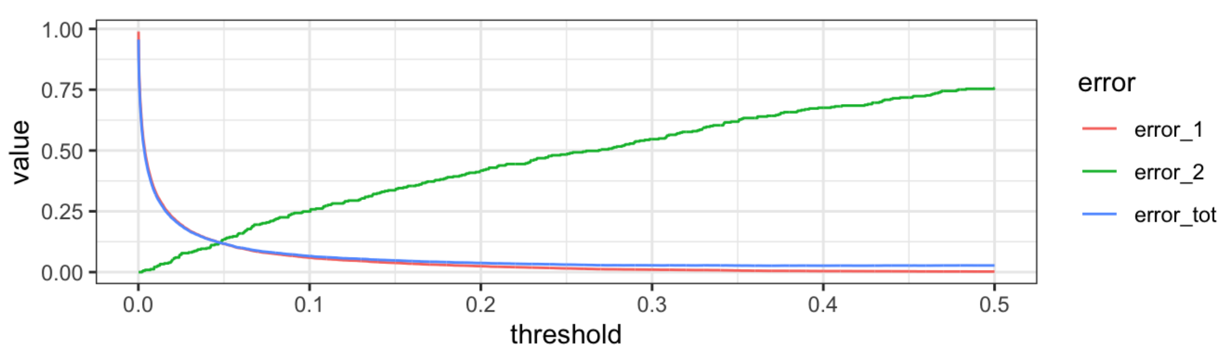
```r
# training data confusion matrix
table(predict(lda_fit)$class, Default$default)
```

```
##
##          No  Yes
##   No   9644  252
##   Yes    23   81
```

Why does the LDA classifier do such a poor job of classifying the customers who default?

```r
table(predict(lda_fit)$posterior[, "Yes"] > 0.2, Default$default)
```

```
##
##            No  Yes
##   FALSE 9432  138
##   TRUE   235  195
```



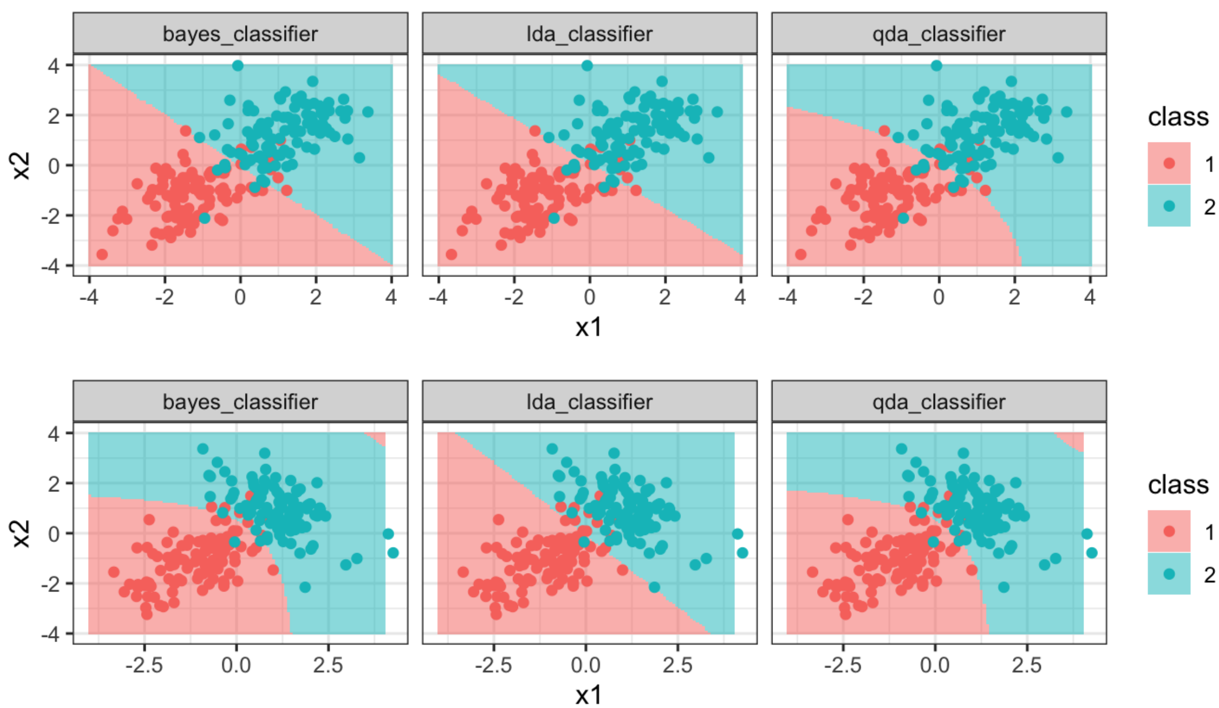
3.4 QDA

LDA assumes that the observations within each class are drawn from a multivariate Gaussian distribution with a class-specific mean vector and a common covariance matrix across all K classes.

Quadratic Discriminant Analysis (QDA) also assumes the observations within each class are drawn from a multivariate Gaussian distribution with a class-specific mean vector but now each class has its own covariance matrix.

Under this assumption, the Bayes classifier assignes observation $X = x$ to class k for whichever k maximizes

When would we prefer QDA over LDA?

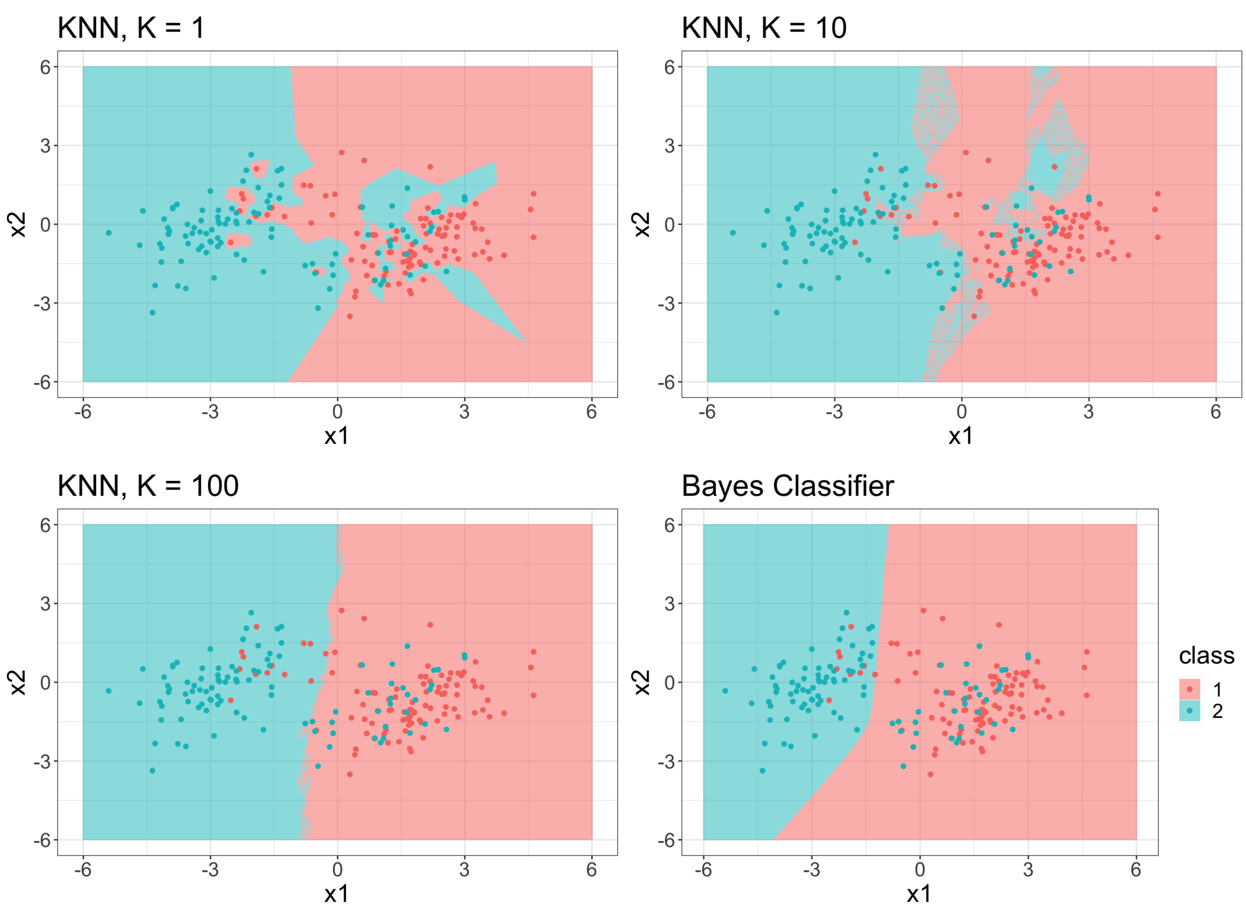


4 KNN

Another method we can use to estimate $P(Y = k|X = x)$ (and thus estimate the Bayes classifier) is through the use of K-nearest neighbors.

The KNN classifier first identifies the K points in the training data that are closest to the test data point $X = x$, called $\mathcal{N}(x)$.

Just as with regression tasks, the choice of K (neighborhood size) has a drastic effect on the KNN classifier obtained.



5 Comparison

LDA vs. Logistic Regression

(LDA & Logistic Regression) vs. KNN

QDA